Scalar $f_0(980)$ meson effect in radiative $\phi \to \pi^+\pi^-\gamma$ decay

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Abstract

We study the effect of scalar-isoscalar $f_0(980)$ meson in the mechanism of the radiative $\phi \to \pi^+\pi^-\gamma$ decay. A phenomenological approach is used to study this decay by considering the contributions of σ-meson, ρ-meson and $f_0(980)$-meson. The interference effects between different contributions are analyzed and the branching ratio for this decay is calculated. We observe that f_0 meson contribution is much larger than the contributions of the other terms.

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I. INTRODUCTION

Radiative decays of vector mesons offer the possibility of investigating new physics features about the interesting mechanism involved in these decays. One particular mechanism involves the exchange of scalar mesons. The scalar mesons, isoscalar σ and $f_0(980)$ and isovector $a_0(980)$, with vacuum quantum numbers $J^{PC} = 0^{++}$ are known to be crucial for a full understanding of the low energy QCD phenomenology and the symmetry breaking mechanisms in QCD. The scalar mesons have been a persistent problem in hadron spectroscopy. In addition to the identification of their nature, the role of scalar mesons in hadronic processes is of extreme importance and the study of radiative decays of vector mesons may provide insights about their role.

In particular, radiative ϕ meson decays, $\phi \to \pi\pi\gamma$ and $\phi \to \pi^0\eta\gamma$, can play a crucial role in the clarification of the structure and properties of scalar $f_0(980)$ and $a_0(980)$ mesons since these decays primarily proceed through processes involving scalar resonances such as $\phi \to f_0(980)\gamma$ and $\phi \to a_0(980)\gamma$, with the subsequent decays into $\pi\pi\gamma$ and $\pi^0\eta\gamma$ [1, 2]. Achasov and Ivanchenko [1] showed that if the $f_0(980)$ and $a_0(980)$ resonances are four-quark ($q^2\bar{q}^2$) states the processes $\phi \to f_0(980)\gamma$ and $\phi \to a_0(980)\gamma$ are dominant and enhance the decays $\phi \to \pi\pi\gamma$ and $\phi \to \pi^0\eta\gamma$ by at least an order of magnitude over the results predicted by the Wess-Zumino terms. Then Close et al. [2] noted that the study of the scalar states in $\phi \to S\gamma$, where $S = f_0\ or\ a_0$, may offer unique insights into the nature of the scalar mesons. They have shown that although the transition rates $\Gamma(\phi \to f_0\gamma)$ and $\Gamma(\phi \to a_0\gamma)$ depend on the unknown dynamics, the ratio of the decay rates $\Gamma(\phi \to a_0\gamma)/\Gamma(\phi \to f_0\gamma)$ provides an experimental test which distinguishes between alternative explanations of their structure. On the experimental side, the Novosibirsk CMD-2 [3, 4] and SND [5] collaborations give the following branching ratios for $\phi \to \pi^+\pi^-\gamma$ and $\phi \to \pi^0\eta\gamma$ decays: $BR(\phi \to \pi^+\pi^-\gamma) = (0.41 \pm 0.12 \pm 0.04) \times 10^{-4}$ [3], $BR(\phi \to \pi^0\eta\gamma) = (0.90 \pm 0.24 \pm 0.10) \times 10^{-4}$ [4], $BR(\phi \to \pi^0\eta\gamma) = (0.88 \pm 0.14 \pm 0.09) \times 10^{-4}$ [5], where the first error is statistical and the second one is systematic.

Theoretically, the role of $f_0(980)$-meson in the radiative decay processes $\phi \to \pi\pi\gamma$ was also investigated by Achasov et al. [6]. They calculated the branching ratio for this decay by considering only $f_0(980)$-meson contribution. They used two different models of $f_0(980)$-meson: the four-quark model and $K\bar{K}$ molecular model. In the four-quark model they obtained the value for the branching ratio as $BR(\phi \to f_0\gamma \to \pi\pi\gamma) = 2.3 \times 10^{-4}$ and in case of the $K\bar{K}$ molecular model, the branching ratio was $BR(\phi \to f_0\gamma \to \pi\pi\gamma) = 1.7 \times 10^{-5}$. Later, Marco et al. considered the radiative ϕ meson decays [7] as well as other radiative vector meson decays within the framework of chiral unitary theory developed earlier by Oller [8]. They obtained the result $BR(\phi \to \pi^+\pi^-\gamma) = 1.6 \times 10^{-4}$ for the branching ratio of the $\phi \to \pi^+\pi^-\gamma$ decay and emphasized that the branching ratio for $\phi \to \pi^+\pi^-\gamma$ decay is twice the one for $\phi \to \pi^0\pi^0\gamma$ decay. Recently, the radiative $\phi \to \pi^0\pi^0\gamma$ decay, where the scalar $f_0(980)$-meson plays an important role was studied by Gökalp and Yılmaz [9] within the framework of a phenomenological approach in which the contributions of σ-meson, ρ-meson and f_0-meson are considered. They analyzed the interference effects between different contributions. Their analysis showed that $f_0(980)$-meson amplitude makes a substantial contribution to the branching ratio of this decay. Furthermore, recently Escribano has been studied the scalar meson exchange in $V \to \pi^0\pi^0\gamma$ decays [10]. He discussed the scalar contributions to the $\phi \to \pi^0\pi^0\gamma$, $\phi \to \pi^0\eta\gamma$ and $\rho^0 \to \pi^0\pi^0\gamma$ decays in the framework of the linear sigma model ($L\sigma M$). He obtained the result $BR(\phi \to \pi^0\pi^0\gamma) = 1.16 \times 10^{-4}$ for the branching ratio of the $\phi \to \pi^0\pi^0\gamma$ decay and noted that, the branching ratio for this decay is dominated by $f_0(980)$ meson amplitude.

In this work, we study the radiative vector meson decay $\phi \to \pi^+\pi^-\gamma$ to investigate the role of the scalar $f_0(980)$ meson and to extract the relevant information on the properties of this scalar meson. Theoretically, the radiative $\phi \to \pi^+\pi^-\gamma$ decay has not been studied extensively up to now. One of the rare studies of this decay was by Marco et al. [7] who neglected the contributions coming from intermediate vector meson states. Therefore, this decay should be reconsidered and the VMD amplitude should be added to the f_0-meson and σ-meson amplitudes.

II. FORMALISM

We study the radiative decay $\phi \to \pi^+\pi^-\gamma$ within the framework of a phenomenological approach in which the contributions of σ-meson, ρ-meson and f_0-meson are considered and we do not make any assumption about the structure of the f_0 meson. In our phenomenological approach we describe the ϕKK-vertex by the effective Lagrangian

$$\mathcal{L}^{eff.}_{\phi KK} = -ig_{\phi KK}\phi^{\mu}\left(K^+\partial_\mu K^- - K^-\partial_\mu K^+\right) \quad , \tag{1}$$

and for the f_0KK-vertex we use the phenomenological Lagrangian

$$\mathcal{L}^{eff.}_{f_0KK} = g_{f_0KK}M_{f_0}K^+K^-f_0 \quad . \tag{2}$$

The effective Lagrangians for the ϕKK- and f_0KK-vertices also serve to define the coupling constants $g_{\phi KK}$ and

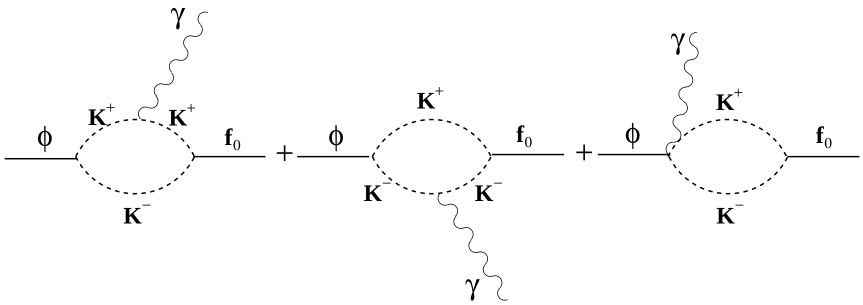

FIG. 1: Feynman diagrams for the decay $\phi \to f_0\gamma$

g_{f_0KK} respectively. The decay width for the $\phi \to K^+K^-$ decay is obtained from the Lagrangian given in Eq. 1 and this decay width is

$$\Gamma(\phi \to K^+K^-) = \frac{g^2_{\phi KK}}{48\pi} M_\phi \left[1 - \left(\frac{2M_K}{M_\phi}\right)^2\right]^{3/2} \quad . \tag{3}$$

We then obtain the coupling constant $g_{\phi KK}$ from the experimental partial width [11] of the radiative decay $\phi \to K^+K^-$ as $g_{\phi KK} = (4.59 \pm 0.05)$. The amplitude of the radiative decay $\phi \to f_0\gamma$ is obtained from the diagrams shown in Fig. 1 where the last diagram assures gauge invariance [1, 12]. This amplitude is

$$\mathcal{M}\left(\phi \to f_0\gamma\right) = -\frac{1}{2\pi^2 M_K^2}\left(g_{f_0KK}M_{f_0}\right)\left(eg_{\phi KK}\right) I(a,b)\left[\epsilon \cdot u \; k \cdot p - \epsilon \cdot p \; k \cdot u\right] \tag{4}$$

where (u, p) and (ϵ, k) are the polarizations and four-momenta of the ϕ meson and the photon respectively, and also $a = M^2_\phi/M^2_K$, $b = M^2_{f_0}/M^2_K$. The $I(a, b)$ function has been calculated in different contexts [2, 8, 13] and is defined as

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f(\frac{1}{b}) - f(\frac{1}{a})\right] + \frac{a}{(a-b)^2}\left[g(\frac{1}{b}) - g(\frac{1}{a})\right] \quad , \tag{5}$$

where

$$f(x) = \begin{cases} -\left[\arcsin(\frac{1}{2\sqrt{x}})\right]^2 \quad , & x > \frac{1}{4} \\ \frac{1}{4}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right]^2 \quad , & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}} \arcsin(\frac{1}{2\sqrt{x}}) \;, & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right] \;, & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x}\left[1 \pm (1-4x)^{\frac{1}{2}}\right] \; . \tag{6}$$

Then, the decay rate for the $\phi \to f_0\gamma$ decay is

$$\Gamma(\phi \to f_0\gamma) = \frac{\alpha}{6(2\pi)^4}\frac{M^2_\phi - M^2_{f_0}}{M^3_\phi} g^2_{\phi KK}\left(g_{f_0KK}M_{f_0}\right)^2 \left|(a-b)I(a,b)\right|^2 \quad . \tag{7}$$

Utilizing the experimental value for the branching ratio $BR(\phi \to f_0\gamma) = (3.4 \pm 0.4) \times 10^{-4}$ for the decay $\phi \to f_0\gamma$ [11], we determine the coupling constant g_{f_0KK} as $g_{f_0KK} = (4.13 \pm 1.42)$. In our calculation, we assume that the radiative decay $\phi \to \pi^+\pi^-\gamma$ proceeds through the reactions $\phi \to \sigma\gamma \to \pi^+\pi^-\gamma$, $\phi \to \rho^\mp\pi^\pm \to \pi^+\pi^-\gamma$ and $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$. Therefore, our calculation is based on the Feynman diagrams shown in Fig. 2. For the $\phi\sigma\gamma$-vertex, we use the effective Lagrangian

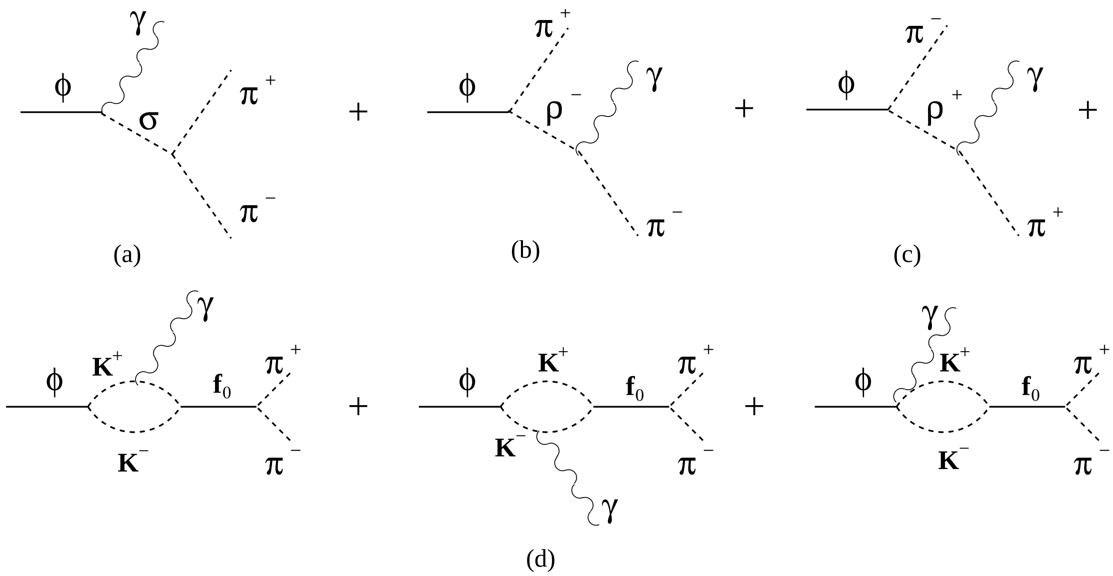

FIG. 2: Feynman diagrams for the decay $\phi \to \pi^+\pi^-\gamma$

$$\mathcal{L}^{eff.}_{\phi\sigma\gamma} = \frac{e}{M_\phi} g_{\phi\sigma\gamma}[\partial^\alpha \phi^\beta \partial_\alpha A_\beta - \partial^\alpha \phi^\beta \partial_\beta A_\alpha]\sigma \quad , \tag{8}$$

which also defines the coupling constant $g_{\phi\sigma\gamma}$. The coupling constant $g_{\phi\sigma\gamma}$ is determined by Gökalp and Yılmaz [9] as $g_{\phi\sigma\gamma} = (0.025 \pm 0.009)$ using the experimental value of the branching ratio for the radiative decay $\phi \to \pi^0\pi^0\gamma$ [14]. For the $\sigma\pi\pi$-vertex we use the effective Lagrangian

$$\mathcal{L}^{eff.}_{\sigma\pi\pi} = \frac{1}{2} g_{\sigma\pi\pi} M_\sigma \vec{\pi} \cdot \vec{\pi}\sigma \quad . \tag{9}$$

The decay width of the σ-meson that results from this effective Lagrangian is given as

$$\Gamma_\sigma \equiv \Gamma(\sigma \to \pi\pi) = \frac{g^2_{\sigma\pi\pi}}{4\pi} \frac{3M_\sigma}{8} \left[1 - \left(\frac{2M_\pi}{M_\sigma}\right)^2\right]^{1/2} \quad . \tag{10}$$

For given values of M_σ and Γ_σ, we use this expression to determine the coupling constant $g_{\sigma\pi\pi}$. Therefore, using the experimental values for M_σ and Γ_σ [15], given as $M_\sigma = (478 \pm 17)\ MeV$ and $\Gamma_\sigma = (324 \pm 21)\ MeV$, we obtain the coupling constant $g_{\sigma\pi\pi} = (5.25 \pm 0.32)$. The $\phi\rho\pi$-vertex is conventionally described by the effective Lagrangian

$$\mathcal{L}^{eff.}_{\phi\rho\pi} = \frac{g_{\phi\rho\pi}}{M_\phi} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \phi_\nu \partial_\alpha \vec{\rho_\beta} \cdot \vec{\pi} \quad . \tag{11}$$

The coupling constant $g_{\phi\rho\pi}$ is calculated as $g_{\phi\rho\pi} = (0.811 \pm 0.081)\ GeV^{-1}$ by Achasov and Gubin [16] using the data on the decay $\phi \to \rho\pi \to \pi^+\pi^-\pi^0$ [11]. For the $\rho\pi\gamma$-vertex the effective Lagrangian

$$\mathcal{L}^{eff.}_{\rho\pi\gamma} = \frac{e}{M_\rho} g_{\rho\pi\gamma} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \vec{\rho_\nu} \cdot \vec{\pi}\ \partial_\alpha A_\beta \quad , \tag{12}$$

is used. At present there is a discrepancy between the experimental widths of the $\rho^0 \to \pi^0\gamma$ and $\rho^+ \to \pi^+\gamma$ decays. We use the experimental rate for the decay $\rho^0 \to \pi^0\gamma$ [11] to extract the coupling constant $g_{\rho\pi\gamma}$ as $g_{\rho\pi\gamma} = (0.69 \pm 0.35)$ since the experimental value for the decay rate of $\phi \to \pi^0\pi^0\gamma$ was used by Gökalp and Yılmaz [9] to estimate the coupling constant $g_{\phi\sigma\gamma}$. Finally, the $f_0\pi\pi$-vertex is described conventionally by the effective Lagrangian

$$\mathcal{L}^{eff.}_{f_0\pi\pi} = \frac{1}{2} g_{f_0\pi\pi} M_{f_0} \vec{\pi} \cdot \vec{\pi} f_0 \quad . \tag{13}$$

In our calculation of the invariant amplitude, we make the replacement $q^2 - M^2 \to q^2 - M^2 + iM\Gamma$, where q and M are four-momentum and mass of the virtual particles respectively, in ρ-, σ- and f_0- propagators in order to take into account the finite widths of these unstable particles and use the experimental value $\Gamma_\rho = (150.2 \pm 0.8)\ MeV$ [11] for ρ-meson. However, since the mass $M_{f_0} = 980\ MeV$ of f_0-meson is very close to the K^+K^- threshold this gives rise to a strong energy dependence on the width of the f_0-meson and to include this energy dependence different expressions for Γ_{f_0} can be used. First option is to use an energy dependent width for f_0

$$\Gamma_{f_0}(q^2) = \Gamma^{f_0}_{\pi\pi}(q^2)\ \theta\left(\sqrt{q^2} - 2M_\pi\right) + \Gamma^{f_0}_{K\overline{K}}(q^2)\ \theta\left(\sqrt{q^2} - 2M_K\right) \quad , \tag{14}$$

where q^2 is the four-momentum square of the virtual f_0-meson and the width $\Gamma^{f_0}_{\pi\pi}(q^2)$ is given as

$$\Gamma^{f_0}_{\pi\pi}(q^2) = \Gamma^{f_0}_{\pi\pi}\ \frac{M^2_{f_0}}{q^2}\sqrt{\frac{q^2 - 4M^2_\pi}{M^2_{f_0} - 4M^2_\pi}} \quad . \tag{15}$$

We use the experimental value for $\Gamma^{f_0}_{\pi\pi}$ as $\Gamma^{f_0}_{\pi\pi} = 40 - 100\ MeV$ [11]. The width $\Gamma^{f_0}_{K\overline{K}}(q^2)$ is given by a similar expression as for $\Gamma^{f_0}_{\pi\pi}(q^2)$. Another and widely accepted option is the work of Flatté [17]. In his work, the expression for $\Gamma^{f_0}_{K\overline{K}}(q^2)$ is extended below the $K\overline{K}$ threshold where $\sqrt{q^2 - 4M^2_K}$ is replaced by $i\sqrt{4M^2_K - q^2}$ so $\Gamma^{f_0}_{K\overline{K}}(q^2)$ becomes purely imaginary. However in our work, we take into account both options. The invariant amplitude $\mathcal{M}(E_\gamma, E_1)$ is expressed as $\mathcal{M}(E_\gamma, E_1) = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d$ where $\mathcal{M}_a$, $\mathcal{M}_b$, $\mathcal{M}_c$ and $\mathcal{M}_d$ are the invariant amplitudes resulting from the diagrams (a), (b), (c) and (d) in Fig. 2 respectively. Therefore, the interference between different reactions contributing to the decay $\phi \to \pi^+\pi^-\gamma$ is taken into account. The differential decay probability for an unpolarized ϕ-meson at rest is given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\ \frac{1}{8M_\phi}\ \mid \mathcal{M} \mid^2 \quad , \tag{16}$$

where E_γ and E_1 are the photon and pion energies respectively. We perform an average over the spin states of ϕ-meson and a sum over the polarization states of the photon. The decay width $\Gamma(\phi \to \pi^+\pi^-\gamma)$ is then obtained by integration

$$\Gamma = \int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \quad , \tag{17}$$

where the minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = (M^2_\phi - 4M^2_\pi)/2M_\phi = 471.8\ MeV$. The maximum and minimum values for the pion energy E_1 are given by

$$\begin{aligned} &\frac{1}{2(2E_\gamma M_\phi - M^2_\phi)}[-2E^2_\gamma M_\phi + 3E_\gamma M^2_\phi - M^3_\phi \\ &\qquad \pm E_\gamma\sqrt{(-2E_\gamma M_\phi + M^2_\phi)(-2E_\gamma M_\phi + M^2_\phi - 4M^2_\pi)}\ ] \quad . \end{aligned} \tag{18}$$

III. RESULTS AND DISCUSSION

In order to determine the coupling constant $g_{f_0\pi\pi}$, we choose for the f_0-meson parameters the values $M_{f_0} = 980\ MeV$ and $\Gamma_{f_0} = (70 \pm 30)\ MeV$. Therefore, through the decay rate that results from the effective Lagrangian given in Eq. 13 we obtain the coupling constant $g_{f_0\pi\pi}$ as $g_{f_0\pi\pi} = (1.58 \pm 0.30)$. If we use the form for $\Gamma^{f_0}_{K\bar{K}}(q^2)$, proposed by Flatté [17], the desired enhancement in the invariant mass spectrum appears in its central part rather than around the f_0 pole. Bramon et al. [18] also encountered a similar problem in their study of the effects of the $a_0(980)$ meson in the $\phi \to \pi^0\eta\gamma$ decay. Therefore, in the analysis which we present below for $\Gamma_{f_0}(q^2)$ we use the form given in Eq. 14. The invariant mass distribution $dB/dM_{\pi\pi} = (M_{\pi\pi}/M_\phi)dB/dE_\gamma$ for the radiative decay $\phi \to \pi^+\pi^-\gamma$ is plotted in Fig. 3 as a function of the invariant mass $M_{\pi\pi}$ of $\pi^+\pi^-$ system. In this figure we indicate the contributions coming from different reactions $\phi \to \sigma\gamma \to \pi^+\pi^-\gamma$, $\phi \to \rho^\mp\pi^\pm \to \pi^+\pi^-\gamma$ and $\phi \to f_0\gamma \to \pi^+\pi^-\gamma$ as well as the contribution of the total amplitude which includes the interference terms as well. It is clearly seen from Fig. 3 that the spectrum for the decay $\phi \to \pi^+\pi^-\gamma$ is dominated by the f_0-amplitude. On the other hand the contribution coming from σ-amplitude can only be noticed in the region $M_{\pi\pi} < 0.7\ GeV$ through interference effects. Likewise ρ-meson contribution can be seen in the region $M_{\pi\pi} < 0.8\ GeV$ so we can say that the contribution of the f_0-term is much larger than the contributions of the σ-term and ρ-term as well as the contribution of the total interference

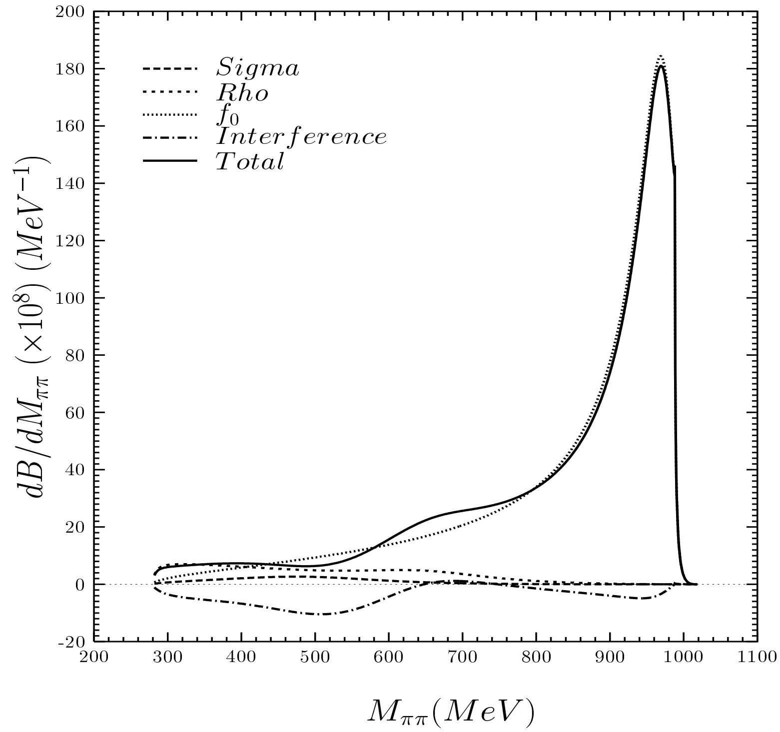

FIG. 3: The $\pi\pi$ invariant mass spectrum for the decay $\phi \to \pi^+\pi^-\gamma$. The contributions of different terms are indicated.

term having opposite sign. The dominant f_0-term characterizes the invariant mass distribution in the region where $M_{\pi\pi} > 0.7\ GeV$. In our study contributions of different amplitudes to the branching ratio of the decay $\phi \to \pi^+\pi^-\gamma$ are $BR(\phi \to f_0\gamma \to \pi^+\pi^-\gamma) = 2.54 \times 10^{-4}$, $BR(\phi \to \sigma\gamma \to \pi^+\pi^-\gamma) = 0.07 \times 10^{-4}$, $BR(\phi \to \rho^\mp\pi^\pm \to \pi^+\pi^-\gamma) = 0.26 \times 10^{-4}$, $BR(\phi \to (f_0\gamma + \pi^\pm\rho^\mp) \to \pi^+\pi^-\gamma) = 2.74 \times 10^{-4}$, $BR(\phi \to (f_0\gamma + \sigma\gamma) \to \pi^+\pi^-\gamma) = 2.29 \times 10^{-4}$ and for the total interference term $BR(\text{interference}) = -0.29 \times 10^{-4}$. We then calculate the total branching ratio as $BR(\phi \to \pi^+\pi^-\gamma) = 2.57 \times 10^{-4}$. Our result is twice the theoretical result for $\phi \to \pi^0\pi^0\gamma$ decay, obtained by Gökalp and Yılmaz [9], as it should be. They obtained the following values: $BR(\phi \to f_0\gamma \to \pi^0\pi^0\gamma) = 1.29 \times 10^{-4}$, $BR(\phi \to \sigma\gamma \to \pi^0\pi^0\gamma) = 0.04 \times 10^{-4}$, $BR(\phi \to \rho^0\pi^0 \to \pi^0\pi^0\gamma) = 0.14 \times 10^{-4}$, $BR(\phi \to (f_0\gamma + \pi^0\rho^0) \to \pi^0\pi^0\gamma) = 1.34 \times 10^{-4}$, $BR(\phi \to (f_0\gamma + \sigma\gamma) \to \pi^0\pi^0\gamma) = 1.16 \times 10^{-4}$ and $BR(\text{interference}) = -0.25 \times 10^{-4}$. Moreover, our calculation for the branching ratio of the radiative decay $\phi \to \pi^+\pi^-\gamma$ is nearly twice the value for the branching ratio of the radiative decay $\phi \to \pi^0\pi^0\gamma$ that was obtained by Achasov and Gubin [16]. Besides, $\phi \to \pi^+\pi^-\gamma$ decay was considered by Marco et al. [7] in the framework of unitarized chiral perturbation theory. The branching ratio for $\phi \to \pi^+\pi^-\gamma$, they obtained, was $BR(\phi \to \pi^+\pi^-\gamma) = 1.6 \times 10^{-4}$ and for $\phi \to \pi^0\pi^0\gamma$ was $BR(\phi \to \pi^0\pi^0\gamma) = 0.8 \times 10^{-4}$. As we mentioned above, they noted that the branching ratio for $\phi \to \pi^0\pi^0\gamma$ is one half of $\phi \to \pi^+\pi^-\gamma$. Therefore our calculation for the branching ratio of $\phi \to \pi^+\pi^-\gamma$ decay is in accordance with the theoretical expectations. A similar relation can be seen between the decay rates of $\omega \to \pi^+\pi^-\gamma$ and $\omega \to \pi^0\pi^0\gamma$ [19]. It was noticed that $\Gamma(\omega \to \pi^0\pi^0\gamma) = 1/2\Gamma(\omega \to \pi^+\pi^-\gamma)$ and the factor 1/2 is a result of charge conjugation invariance to order α which imposes pion pairs of even angular momentum. The experimental value of the branching ratio for $\phi \to \pi^+\pi^-\gamma$, measured by Akhmetshin et al., is $BR(\phi \to \pi^+\pi^-\gamma) = (0.41 \pm 0.12 \pm 0.04) \times 10^{-4}$ [3]. So the value of the branching ratio that we obtained is approximately six times larger than the value of the measured branching ratio. As it was stated by Marco et al. [7], we should not compare our calculation for the branching ratio of the radiative decay $\phi \to \pi^+\pi^-\gamma$ directly with experiment since the experiment is done using the reaction $e^+e^- \to \phi \to \pi^+\pi^-\gamma$, which interferes with the off-shell ρ dominated amplitude coming from the reaction $e^+e^- \to \rho \to \pi^+\pi^-\gamma$ [20]. Also the result in [3] is based on model dependent assumptions.

IV. ACKNOWLEDGEMENT

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